

## Calculus AB Derivatives & Integral Rules

### Properties:

$$\frac{d}{dx}(c) = 0 \quad (c \cdot f)' = c \cdot f' \quad (f \pm g)' = f' \pm g'$$

$$\frac{d}{dx} \left( \int f(x) dx \right) = f(x) \quad \int c \cdot f(x) dx = c \int f(x) dx$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

### Derivative Rules:

Product:

$$(f \cdot g)' = f \cdot g' + g \cdot f'$$

Quotient:

$$\left( \frac{f}{g} \right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$$

Chain Rule (Composition of functions):

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, \text{ or } \frac{d}{dx}(f(u)) = f'(u) \cdot \frac{du}{dx}, \text{ or } \frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

**Integration:** Use Algebra, Trigonometry, or u-substitution to get the integral into one of the patterns below.

### Derivatives & Integrals:

Chain rule

$$\frac{d}{dx}(u^n) = n \cdot u^{n-1} \frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$$

$$\frac{d}{dx}(\ln|u|) = \frac{1}{u} \frac{du}{dx}$$

Integral(Antiderivative)

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$

$$\int \cos u du = \sin u + c$$

$$\int \sin u du = -\cos u + c$$

$$\int \sec^2 u du = \tan u + c$$

$$\int \csc^2 u du = -\cot u + c$$

$$\int \sec u \tan u du = \sec u + c$$

$$\int \csc u \cot u du = -\csc u + c$$

$$\int e^u du = e^u + c$$

$$\int \frac{du}{u} = \ln|u| + c$$

$$\frac{d}{dx}(\log_a |u|) = \frac{1}{u \ln a} \frac{du}{dx}$$

$$\frac{d}{dx}(a^u) = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{d}{dx}(\cot^{-1} u) = -\frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$\frac{d}{dx}(\csc^{-1} u) = -\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$\int a^u du = \frac{a^u}{\ln a} + c$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{du}{|u|\sqrt{u^2-a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{u}{a}\right| + c$$

$$\int \tan u du = \ln|\sec u| + c$$

$$\int \cot u du = \ln|\sin u| + c$$

$$\int \sec u du = \ln|\sec u + \tan u| + c$$

$$\int \csc u du = \ln|\csc u - \cot u| + c$$

## Calculus AB often used formulas

1) Calculator in Radian Mode

2) Continuity and Differentiability:

$$f(x) = \begin{cases} g(x), & x < a \\ h(x), & x \geq a \end{cases} \quad \text{then} \quad f'(x) = \begin{cases} g'(x), & x < a \\ h'(x), & x > a \end{cases}$$

$$\text{Continuity: } \lim_{x \rightarrow a^-} g(x) \stackrel{?}{=} h(a) \stackrel{?}{=} \lim_{x \rightarrow a^+} h(x)$$

$$\text{Differentiability: Continuity plus } \lim_{x \rightarrow a^-} g'(x) \stackrel{?}{=} \lim_{x \rightarrow a^+} h'(x)$$

$$3) \text{ Area} = \int_a^b \text{height} \, dx, \quad \text{Volume} = \int_a^b \text{Area} \, dx$$

4) Relative Max/Min of  $k(x) \rightarrow k'(x) = 0$  or und (critical points)

Max  $\rightarrow k'$  changes from + to -

Min  $\rightarrow k'$  changes from - to +

5) Absolute Max/Min (max/min) of  $k(x)$  @ critical points or endpoints

6) Points of inflection of  $k(x) \rightarrow k''(x) = 0$  or und and  $k''$  changes sign

$$7) \text{ Average value: } f_{ave} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

$$\text{Application of average value: } v_{ave} = \frac{1}{b-a} \int_a^b v(t) \, dt = \frac{s(b) - s(a)}{b-a}$$

$$8) \text{ Position-Velocity-Acceleration: } \begin{aligned} v(t) = s'(t) &\quad \rightarrow \quad \int v(t) \, dt = s(t) + c \\ a(t) = v'(t) = s''(t) &\quad \rightarrow \quad \int a(t) \, dt = v(t) + c \end{aligned}$$

$$9) \text{ Initial condition formula: } (x_0, y_0), \quad y(x) = y_0 + \int_{x_0}^x y'(t) \, dt$$

$$\text{Application } (t_0, s_0), \quad s(t) = s_0 + \int_{t_0}^t v(x) \, dx$$

10) Distance- Displacement:

$$\text{Distance} = \int_a^b |v(t)| dt \rightarrow \text{area}$$

$$\text{Displacement} = \int_a^b v(t) dt \rightarrow \text{net signed area}$$

11) Fundamental Theorem of Calculus

$$\int_a^b f'(x) dx = f(b) - f(a)$$

$$\frac{d}{dx} \int_a^{f(x)} h(t) dt = h(f(x)) \cdot f'(x)$$

12) Given a graph of a function f,

$$\text{and } g(x) = \int_a^x f(t) dt, \text{ then}$$

$$g'(x) = f(x) \text{ make sign line here}$$

$$g''(x) = f'(x) \text{ make sign line here}$$

13) Related rates: find  $\frac{dV}{dt}$ , given  $\frac{dr}{dt}$  and  $V = \pi r^2 h$

Find connection between r and h and substitute ( in cones look for similar triangles).  
Differentiate with respect to t. Don't forget the chain rule.

$$V = \pi r^2 h, \quad h = 2r$$

$$\text{Example: } V = \pi r^2 (2r) = 2\pi r^3$$

$$\frac{dV}{dt} = 2\pi \left( 3r^2 \frac{dr}{dt} \right)$$

14) Slope fields: Slope field given-  $\frac{dy}{dx} = x^2(y-1)$

Solve: find y - separate variables, integrate both sides, only one c is necessary, use initial condition. If 2 branches, use the branch the contains the point given (initial value)

## Trigonometry Functions

**Reciprocal functions**       $\sin \theta = \frac{1}{\csc \theta}$        $\cos \theta = \frac{1}{\sec \theta}$        $\tan \theta = \frac{1}{\cot \theta}$

**Functions in terms of sin & cos**       $\tan \theta = \frac{\sin \theta}{\cos \theta}$        $\cot \theta = \frac{\cos \theta}{\sin \theta}$        $\csc x = \frac{1}{\sin x}$

$\sec x = \frac{1}{\cos x}$

### Negative functions

$\sin(-\theta) = -\sin \theta$  (odd function)

$\cos(-\theta) = \cos \theta$  (even function)

$\tan(-\theta) = -\tan \theta$  (odd function)

### Cofunctions: sine – cosine, etc

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ ,       $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$ ,       $\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$ ,       $\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$

### Pythagorean Identities

$\sin^2 \theta + \cos^2 \theta = 1$       divide by  $\sin^2 \theta \rightarrow 1 + \cot^2 \theta = \csc^2 \theta$       divide by  $\cos^2 \theta \rightarrow \tan^2 \theta + 1 = \sec^2 \theta$

### Cosine: Sum / Difference      →      Cosine double angle

$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$       →       $\cos 2\theta = \cos(\theta + \theta) = \cos^2 \theta - \sin^2 \theta$

### Cosine      →      Double angle      →      Power reduction      →      Half angle

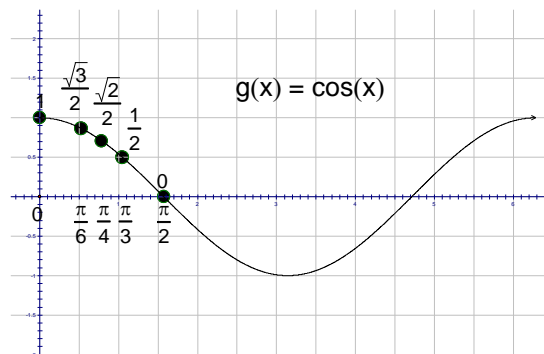
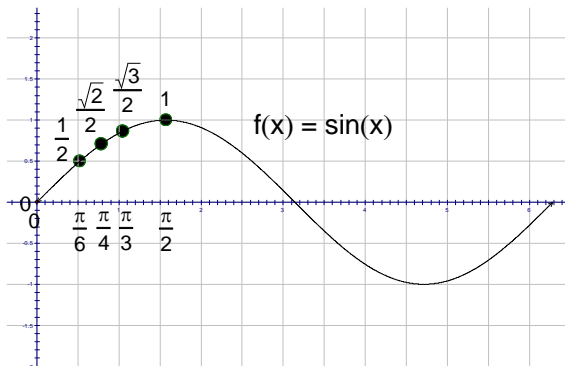
substitute for  $\cos^2 \theta$       →       $\cos 2\theta = 1 - 2\sin^2 \theta$       →       $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$       →       $\sin \theta = \pm \sqrt{\frac{1 - \cos 2\theta}{2}}$

substitute for  $\sin^2 \theta$       →       $\cos 2\theta = 2\cos^2 \theta - 1$       →       $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$       →

$\cos \theta = \pm \sqrt{\frac{1 + \cos 2\theta}{2}}$

### Sine: Sum / Difference      →      Sine double angle

$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$       →       $\sin 2\theta = \sin(\theta + \theta) = 2\sin \theta \cos \theta$



" $\pi$ " is a special # and  $\pi \approx 3.142$

## Exponential and Logarithmic Functions

Exponential and logarithmic with the same base are inverse functions (reflect over the  $y = x$  line)

**Exponential function:**  $f(x) = a^x$ , Domain:  $x \in \mathbb{R}$ , Range:  $y > 0$

$$\lim_{x \rightarrow -\infty} a^x = 0, \quad \lim_{x \rightarrow \infty} a^x = \infty$$

**Properties of Exponential functions:**

$$a^n + a^m = a^n + a^m$$

$$a^n \cdot a^m = a^{n+m}$$

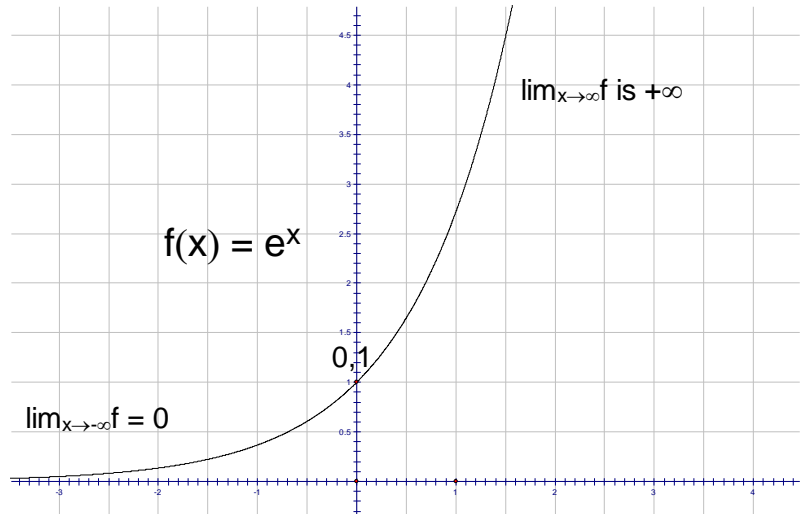
$$(a^n)^m = a^{nm}$$

$$(a^n b^m)^k = a^{kn} b^{km}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$a^0 = 1$$



**Logarithmic function:**  $f(x) = \log_n a$ , Domain:  $x > 0$ , Range:  $y \in \mathbb{R}$

$$\lim_{x \rightarrow 0^+} \log_n a = -\infty, \quad \lim_{x \rightarrow \infty} \log_n a = \infty$$

**Properties of Logarithmic functions:**

$$y = \log_a x \Leftrightarrow x = a^y$$

$$\log_a (nm) = \log_a n + \log_a m$$

$$\log_a \left( \frac{n}{m} \right) = \log_a n - \log_a m$$

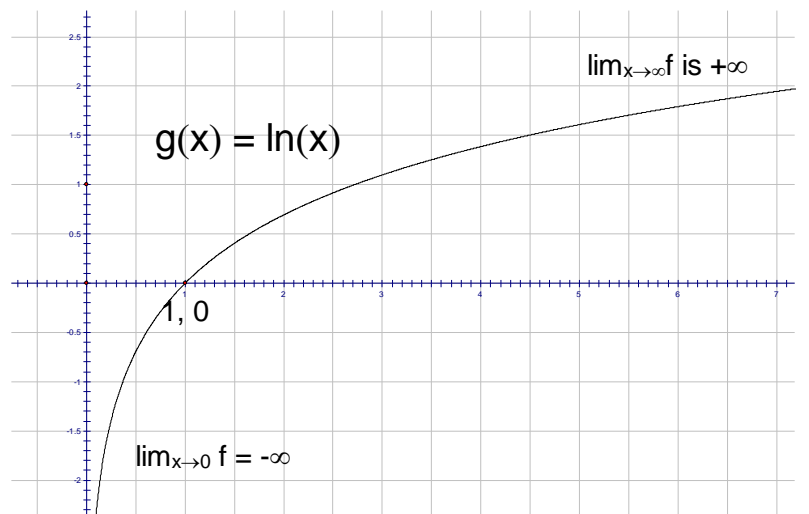
$$\log_a n^m = m \log_a n$$

$$\log_n m = \frac{\log_a m}{\log_a n}$$

$$\log_{10} n \Leftrightarrow \log n$$

$$\log_e n \Leftrightarrow \ln n$$

$$\log_n 1 = 0; \quad \ln e = 1; \quad \log_{10} 10 = 1$$



**Note:**  $\log x$  is understood to be  $\log_{10} x$  and  $\ln x$  is  $\log_e x$

“e” is special base and  $e \approx 2.718$