

## Der – Int – Calc – Thm - Gph Worksheet

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### Derivatives

Basic derivatives:  $(f'(x), \frac{dy}{dx}, y')$  Find  $y'$ .

1)  $y = 4$

2)  $y = 2x^3 + 3x^2 - 5x + 1$

3)  $y = \sin x$

4)  $y = \cos x$

5)  $y = \tan x$

6)  $y = \csc x$

7)  $y = \sec x$

8)  $y = \cot x$

9)  $y = \ln x$

10)  $y = e^x$

Basic derivatives with chain rule:  $y' = operation(u)u'$

1)  $y = (x^2 + 3x - 1)^3$

2)  $y = \sin 4x$

3)  $y = \cos x^2$

4)  $y = \tan(3x + 1)$

5)  $y = \cot 2x$

6)  $y = \sec(3x - 4)$

7)  $y = \csc(x^2 - 1)$

8)  $y = \ln(2x + 3)$

9)  $y = e^{3x^2 - 4}$

**Product rule using basic derivatives:**  $y = fg$ ,  $y' = fg' + gf'$

1)  $y = \ln x \cdot x^2$

2)  $y = \cos x \cdot e^x$

3)  $y = \sin x \cdot \tan x$

4)  $y = \cot x \cdot e^x$

5)  $y = 2x^3 e^x$

6)  $y = \csc x \cdot \sec x$

**Quotient rule using basic derivatives:**  $y = \frac{f}{g}$ ,  $y' = \frac{gf' - fg'}{g^2}$

1)  $y = \frac{\cos x}{e^x}$

2)  $y = \frac{3x^2 - 3x + 1}{\tan x}$

3)  $y = \frac{\ln x}{\sin x}$

4)  $y = \frac{\csc x}{x^4}$

5)  $y = \frac{e^x}{\cot x}$

6)  $y = \frac{\sec x}{\ln x}$

## Special Derivatives

$$1) \frac{d}{dx} f(3x^2) \quad 2) \frac{d}{dx} [f(x) + g(x)] \quad 3) \frac{d}{dx} [f(x) \cdot g(x)] \quad 4) \frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right]$$

## Implicit Differentiation:

$$3) 3xy^2 - 5x + 12y + 2e^y = 5$$

a) Find  $\frac{dy}{dx}$

b) Find  $\frac{dy}{dt}$

## Logarithmic Differentiation:

$$4) y = x^{\sin x}$$

## Fundamental Theorem of Calculus: (derivation of an integral)

$$5) \frac{d}{dx} \left( \int_{2x}^{x^2} t^3 + \cos t \, dt \right)$$

$$6) g(x) = 2x - 7 + \int_1^x f(t) \, dt$$

## Integrals

### Basic Integrals (anti derivatives)

$$\int du = u + c \quad \int k f(u) du = k \int f(u) du$$
$$\int (f(u) \pm g(u)) du = \int f(u) du \pm \int g(u) du$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c \quad \text{Power rule good for any power except } x^{-1} \text{ which is equal to } \frac{1}{x}$$

$$\text{examples: } \int x^3 dx = \frac{x^4}{4} + c, \quad \int x^{-3} dx = \frac{x^{-2}}{-2} + c, \quad \int x^{\frac{1}{2}} dx = \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$1) \int x^7 dx =$$

$$2) \int \sqrt[3]{x} dx =$$

$$3) \int \frac{dx}{x^{-3}} =$$

### U substitution and power rule:

$$4) \int x(2x^2 - 5)^3 dx =$$

$$5) \int \frac{3x^2}{5x^3 - 7} dx =$$

$$\int u^{-1} du \text{ same as } \int \frac{1}{u} du \text{ same as } \int \frac{du}{u} = \ln|u| + c$$

$$6) \int \frac{1}{t} dt =$$

$$7) \int \frac{x dx}{x^2 - 4} =$$

$$8) \int \frac{1}{2y - 3} dy =$$

$$\int e^u du = e^u + c$$

$$9) \int e^x dx =$$

$$10) \int e^{2x} dx =$$

$$11) \int 3xe^{x^2} dx =$$

$$\int \sin u du = -\cos u + c$$

$$12) \int \sin x dx =$$

$$13) \int \sin 2x dx =$$

$$14) \int x \sin x^2 dx =$$

$$\int \cos u du = \sin u + c$$

$$15) \int \cos x dx =$$

$$16) \int -3 \cos 4x dx =$$

$$17) \int x^3 \cos 2x^4 dx =$$

$$\int \sec^2 u \, du \text{ same as } \int (\sec u)^2 \, du = \tan u + c$$

$$18) \int \sec^2 5x \, dx =$$

$$19) \int x^4 (\sec x^5)^2 \, dx =$$

$$\int \csc^2 u \, du \text{ same as } \int (\csc u)^2 \, du = -\cot u + c$$

$$20) \int \csc^2(-3x) \, dx =$$

$$21) \int -2x \csc^2(x^2) \, dx$$

$$\int \sec u \tan u \, du = \sec u + c$$

$$22) \int \sec 5x \tan 5x \, dx$$

$$23) \int x^5 \sec x^6 \tan x^6 \, dx$$

$$\int \csc u \cot u \, du = -\csc u + c$$

$$24) \int \csc 3x \cot 3x \, dx$$

## Calculator

1) Find the zero's: (quadratic formula)

$$f(x) = 2x^2 - 5x - 8$$

2) Find the zero(s) (no quadratic, calculate zero in window)

$$f(x) = 2x^3 - 3x^2 + 2x - 5$$

3) Find the intersection:

Can be written 3 different ways: In 3 you only get the x coordinate (plug in the function to get the y coordinate)

a)  $y = 2x^3 - 4$   
 $y = e^{-x}$

b)  $2x^3 - 4 = e^{-x}$

c)  $0 = e^{-x} - 2x^3 + 4$

4) Limits:

$$\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x}$$

$$\lim_{x \rightarrow -\infty} \left( \frac{x}{2} + \sqrt{\frac{x^2}{4} + x} \right)$$

5) On the interval  $[0, 12]$ , find the area of the region bound by the curves

$$y = 12 \sin\left(\frac{x}{4}\right) \quad \& \quad y = 7.$$

Find points of intersection, store the x values in A & B. Use Math 9, Vars, Y-Vars, etc. and A & B.

6) Given  $v(t) = 12 \sin\left(\frac{x}{4}\right)$ ,  $0 \leq t \leq 20$ ,  $x(0) = -14$

a) Find  $a(7)$ . Use in window use Calc 6 and type in 7.

b) Find position  $x(15)$

c) Find the distance traveled on the interval  $[2, 16]$

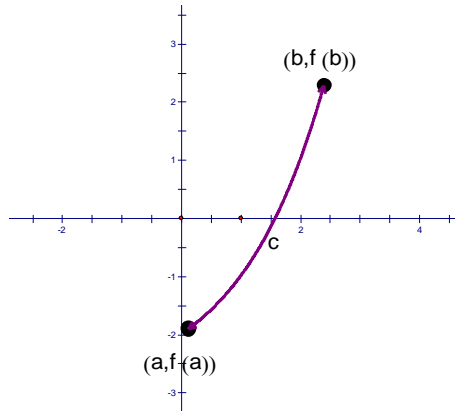
d) Find the displacement on the interval  $[4, 14]$



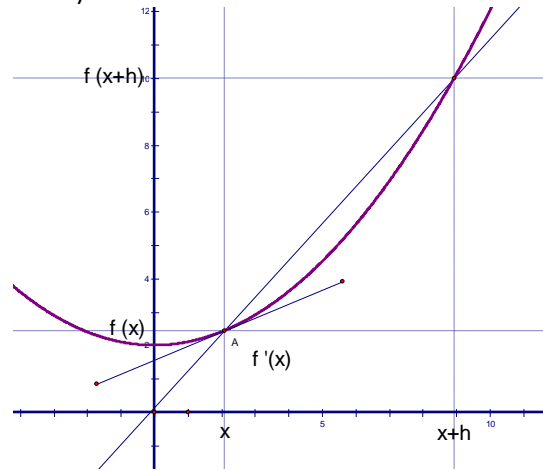
# Theorems – Definitions – Graphs

Write a theorem or definition that each graph is trying to display.

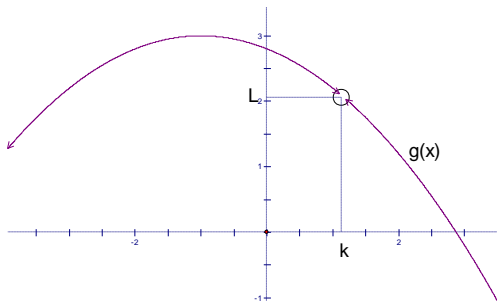
1)



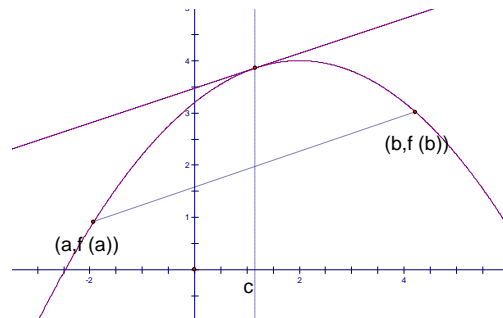
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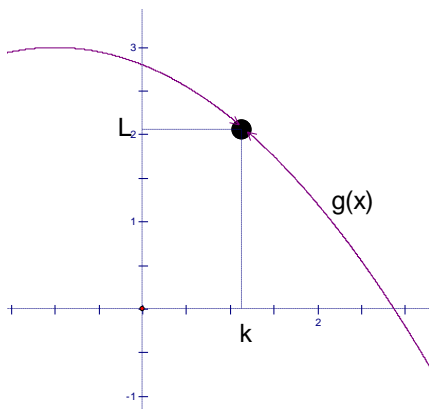
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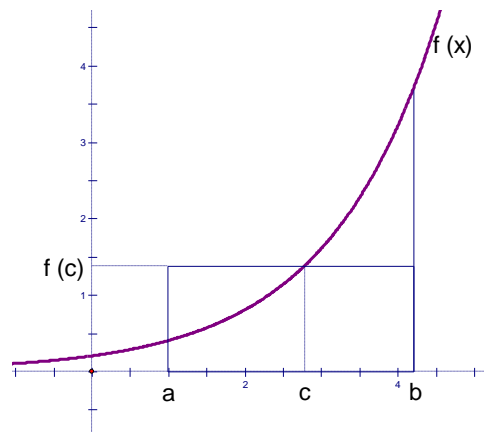
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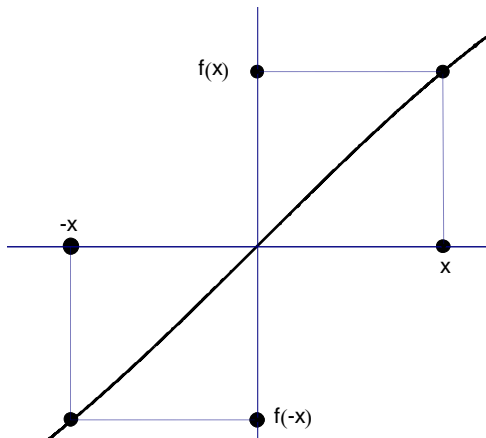
5)



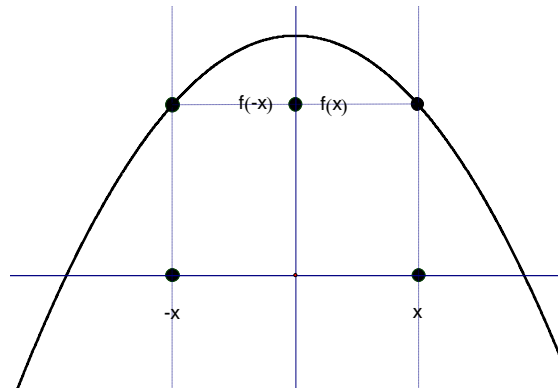
6)



7)



8)

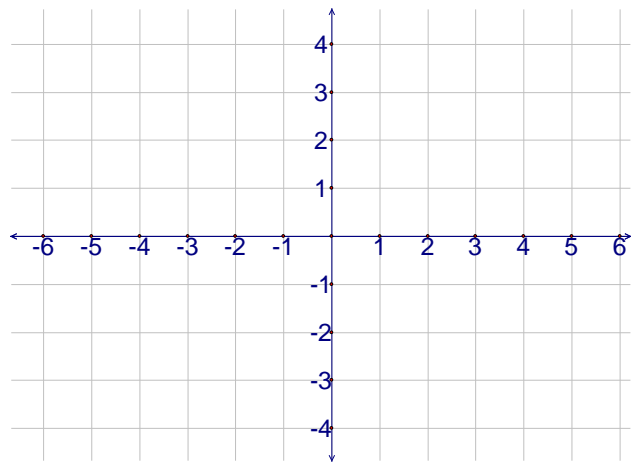


Sketch the function that satisfies the given information.

9) Domain  $[-6, 5]$   $\lim_{x \rightarrow -3^+} f(x) = -1$ ,

$\lim_{x \rightarrow -3^-} f(x) = 4$ ,  $f(-3) = 2$ ,  $\lim_{x \rightarrow 2} f(x) = -1$ ,

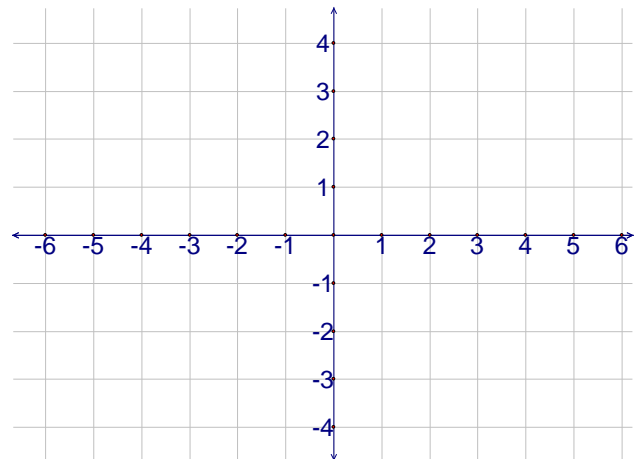
$f(2) = 0$



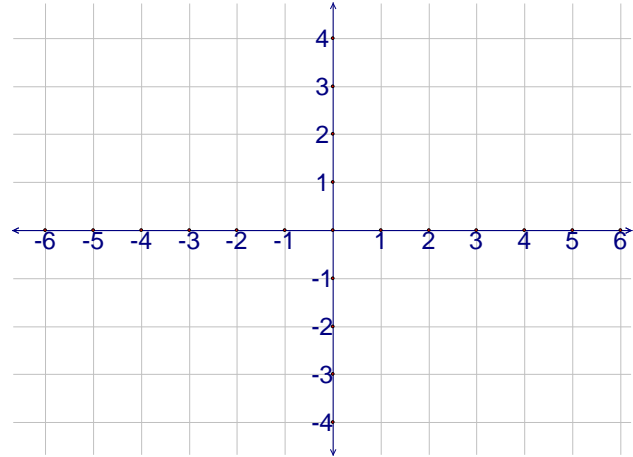
10) Domain  $(-\infty, \infty)$ ,  $f(x)$  is continuous at

$x = -1$ , but not differentiable and  $f(-1) = 3$ . The

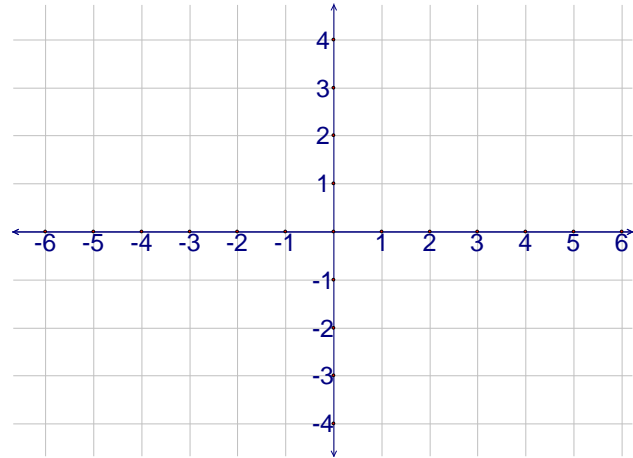
x-intercept is  $-5$  and the y-intercept is  $2$



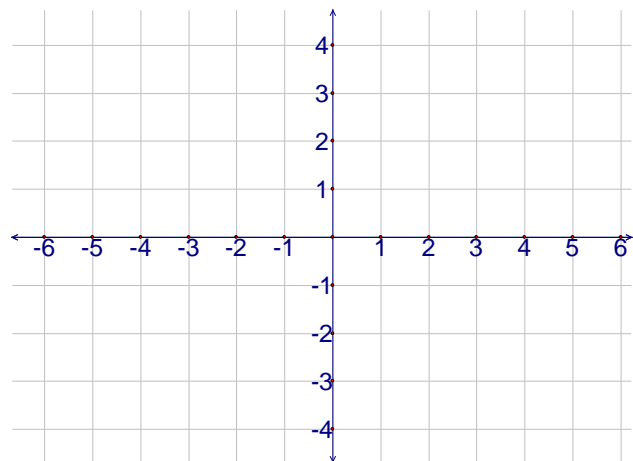
11)  $f(-4)=3$ ,  $f'(-2)=0$ ,  $f'(3)=0$ ,  
 $f'(x)<0$  in the interval  $(-6,-2)$ ,  $f'(x)>0$   
in the intervals  $(-2,3)$  and  $(3,5)$ ,  $f$  is  
continuous on the interval  $[-6,5]$



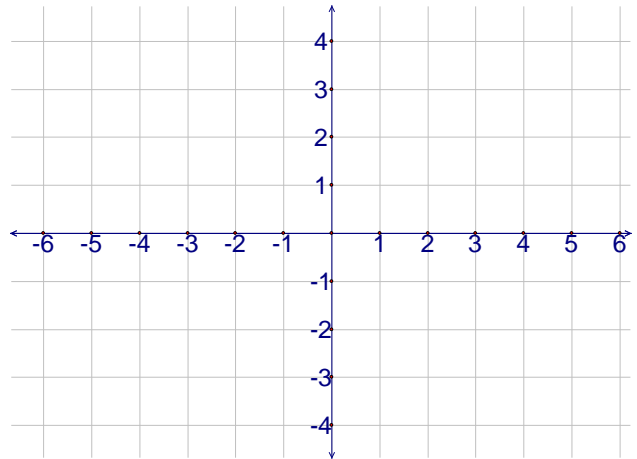
12)  $f$  is continuous on the interval  $(-3,2)$ ,  
 $\lim_{x \rightarrow -3^+} f(x) \rightarrow -\infty$ ,  $\lim_{x \rightarrow 2^-} f(x) \rightarrow \infty$ ,  
 $f''(x)<0$  in the interval  $(-3,-1)$  and  
 $f''(x)>0$  in the interval  $(-1,2)$ .



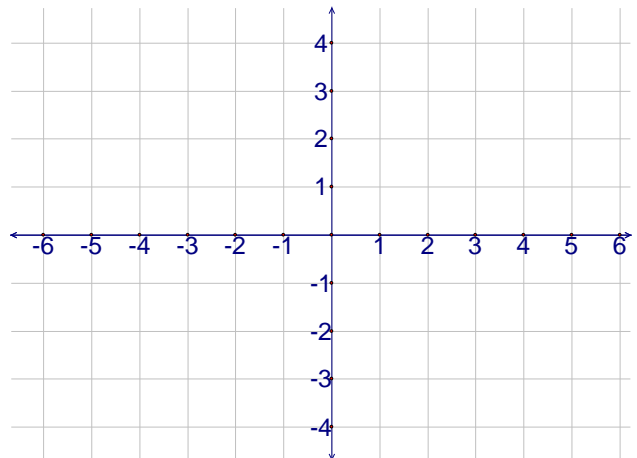
13) The domain of  $f$  is  $[-4,3)$ ,  $f'(x)>0$  for all  
 $x$ ,  $f'(2)=0$ , there is an absolute minimum of  
 $-3$ , but no absolute maximum.



14) The domain of  $f$  is  $(-2, 5)$ , there is a relative (local) minimum at  $x = 0$  and a relative maximum at  $x = 3$  and a point of inflection at  $x = 2$ , but no absolute maximum or absolute minimum



15) The domain of  $f$  is  $[-4, 5]$ ,  $\lim_{x \rightarrow -1^-} f(x) = 2$ ,  $\lim_{x \rightarrow -1^+} f(x) = 4$ ,  $f(-1) = 2$ , on the interval  $(-4, -1)$  both  $f'(x)$  and  $f''(x) > 0$ , on the interval  $(-1, 5)$  both  $f'(x)$  and  $f''(x) < 0$ .



16) The domain of  $f$  is  $[-4, 5]$ , there are 3 critical points at  $x = -2, 1, 3$ , absolute (global) maximum at  $x = -4$ , absolute (global) minimum at  $x = 1$ , relative maximum at  $x = 3$ , but no relative extreme at  $x = -2$ .

