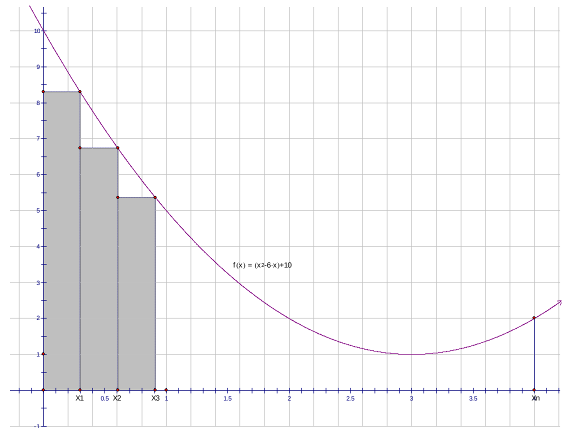
Area under the curve $f\left(x\right)=x^2-6x+10 \quad \left[0,4\right]$ using rectangles from the right



Using "n" rectangles (partitions), therefore $\Delta x=\dfrac{4-0}{n}=\dfrac{4}{n}$

$$A=\Delta x\cdot f\left(x_1\right)+\Delta x\cdot f\left(x_2\right)+\Delta x\cdot f\left(x_3\right)+\ldots\ldots\ldots+\Delta x\cdot f\left(x_n\right)$$

Summing up the areas of "n" rectangles $A=\sum_{k=1}^{n}\Delta x\cdot f\left(x_k\right)$

then letting "n" go to infinity

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\Delta x\cdot f\left(x_k\right)$$

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{4}{n}\left(x_k^{\ 2}-6x_k+10\right), \quad \text{but } x_k=k\cdot\Delta x=k\frac{4}{n}$$

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{4}{n}\left(\left(\frac{4k}{n}\right)^2-6\left(\frac{4k}{n}\right)+10\right)$$

$$\sum_{i=1}^{n}c=cn, \quad \sum_{i=1}^{n}i=\frac{n\left(n+1\right)}{2}, \quad \sum_{i=1}^{n}i^2=\frac{n\left(n+1\right)\left(2n+1\right)}{6}, \quad \sum_{i=1}^{n}i^3=\frac{n^2\left(n+1\right)^2}{4}$$

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{4}{n}\left(\frac{16k^2}{n^2}-\frac{24k}{n}+10\right)$$

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{64k^2}{n^3}-\frac{96k}{n^2}+\frac{40}{n}$$

$$A=\lim_{n\to\infty}\left(\frac{64}{n^3}\sum_{k=1}^{n}k^2-\frac{96}{n^2}\sum_{k=1}^{n}k+\frac{40}{n}\sum_{k=1}^{n}1\right)$$

using closed form of summations

$$A=\lim_{n\to\infty}\left(\frac{64}{n^3}\left(\frac{n\left(n+1\right)\left(2n+1\right)}{6}\right)-\frac{96}{n^2}\left(\frac{n\left(n+1\right)}{2}\right)+\frac{40}{n}n\right)$$

multiplying

$$A=\lim_{n\to\infty}\left[\frac{64}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right)-\frac{96}{n^2}\left(\frac{n^2+n}{2}\right)+40\right]$$

distributing

$$A=\lim_{n\to\infty}\left[\frac{64}{3}+\frac{32}{n}+\frac{32}{3n^2}-48-\frac{48}{n}+40\right]$$

Let $n\to\infty \quad A=\left[\dfrac{64}{3}+-48+40\right]$

$$A=\frac{40}{3}\approx 13.333$$

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$$A=\Delta x\cdot f\left(x_1\right)+\Delta x\cdot f\left(x_2\right)+\Delta x\cdot f\left(x_3\right)+\dots\dots\dots+\Delta x\cdot f\left(x_n\right)$$

Summing up the areas of "n" rectangles $A=\sum_{k=1}^{n}\Delta x\cdot f\left(x_k\right)$ then letting "n" go to infinity

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\Delta x\cdot f\left(x_k\right)$$

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{x}{n}\left(x_k{}^2-6x_k+10\right), \quad \text{but } x_k=k\cdot\Delta x=k\frac{x}{n}$$

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{x}{n}\left(\left(\frac{kx}{n}\right)^2-6\left(\frac{kx}{n}\right)+10\right)$$

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{x}{n}\left(\frac{k^2x^2}{n^2}-\frac{6kx}{n}+10\right)$$

$$A=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^2x^3}{n^3}-\frac{6kx^2}{n^2}+\frac{10x}{n}$$

$$A=\lim_{n\to\infty}\left(\frac{x^3}{n^3}\sum_{k=1}^{n}k^2-\frac{6x^2}{n^2}\sum_{k=1}^{n}k+\frac{10x}{n}\sum_{k=1}^{n}1\right)$$

using closed form of summations

$$A=\lim_{n\to\infty}\left(\frac{x^3}{n^3}\left(\frac{n\left(n+1\right)\left(2n+1\right)}{6}\right)-\frac{6x^2}{n^2}\left(\frac{n\left(n+1\right)}{2}\right)+\frac{10x}{n}n\right)$$

multiplying

$$A=\lim_{n\to\infty}\left[\frac{x^3}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right)-\frac{6x^2}{n^2}\left(\frac{n^2+n}{2}\right)+10x\right]$$

distributing

$$A=\lim_{n\to\infty}\left[\frac{x^3}{3}+\frac{x^3}{2n}+\frac{x^3}{6n^2}-3x^2-\frac{3x^2}{n}+10x\right]$$

Let $n\to\infty$ $A=\left[\frac{x^3}{3}-3x^2+10x\right]$

If $x=4$ then $A=\frac{4^3}{3}-3\cdot4^2+10\cdot4=\frac{64}{3}-48+40=\frac{40}{3}$

If $x=5$ then $A=\frac{5^3}{3}-3\cdot5^2+10\cdot5=\frac{125}{3}-75+50=\frac{50}{3}$